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Introduction to an Alternative Single Frequency Time-Relative Kinematic Positioning Method based on GNSS

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***Abstract* — In this work, a new method of GNSS-based single-frequency determination of real-time relative displacements is introduced, with accuracy somewhat better than float RTK, but worse than the fixed RTK single-frequency solutions. The basic idea is shown through simulations in a simplified two-dimensional context containing the main aspects of a satellite navigation system. The method requires a fixed reference station and a user rover receiver, both supplying code and phase measurements. Although it is desirable to have phase-lock during consecutive measurements, no ambiguity resolution procedure is required. Still, the present approach could be used as a temporary solution before the ambiguities are fixed, replacing the RTK float solutions in cases where the relative displacement from an initial user position is of greater interest than its absolute position, such as in the determination of flight trajectories during maneuvers.**

***Keywords* — GNSS, RTK, trajectory determination**

I. INTRODUCTION

Satellite navigation still has much to be explored. New applications may become feasible as new GNSS-based positioning techniques provide increased performance-cost ratio. Particular applications don't usually require the most sophisticated navigation systems available but only some specific features among them. As an example, in a static surveying application where one has not real time constraints it seems wasteful to use a real-time solution such as RTK (Real Time Kinematic).

Precision real-time relative positioning can be achieved through RTK with centimeter-level accuracy. However, phase ambiguity resolution requires some kind of initialization procedure, which typically takes from minutes to hours, and every time a cycle slip occurs, it has to start over. On the other hand, code-only based implementations, such as DGPS, are ambiguity free, but don't provide enough accuracy for some applications. There is also the so-called float solution, in which ambiguities remain not resolved, but carrier-phase is somehow included in the least-squares algorithm helping position estimation [1]. This typically precedes the fixed solution in professional RTK-ready GPS receivers and has a better accuracy than DGPS.

Sometimes absolute positioning is not necessary, but only the relative displacement from an initial user position. In other words, in these cases it is desirable to obtain the vector from a user starting point to its current position rather than the position of the user at the two instants based on an Earth reference frame. One such example is the measurement of flight trajectories during maneuvers, in case the geographic position or height where the maneuver is performed is not relevant, but the maneuver itself. This kind of positioning will be referred here as time-relative displacement positioning (TRDP).

The obvious way of doing TRDP is by simply making differences between single point positioning solutions to obtain the displacement vectors. Another way is to model the displacement vector itself as an unknown variable and solve for it. For code-only based positioning, those methods are equivalent, and there is no relevant advantage or disadvantage when doing it either way. However, if carrier phase measurements are used for better accuracy, since there is a constant ambiguity in the integer number of wavelengths, it is much simpler to measure displacements by tracking the phase and counting the number of cycle advances than finding the exact number of cycles between receiver and satellites, which would be required to solve for the initial and final positions. This principle is taken in advantage for the proposed method.

References [2]-[3] use so called time-differences to achieve TRDP. In their work, raw data of low cost L1 GPS receivers are used together with external corrections to achieve precision under the sub-decimeter range during time intervals of several minutes. The time-differenced carrier phase observable has the property of cancelling the ambiguities if no cycle-slip occurs as well as partially canceling time correlated error sources such as ionospheric and tropospheric delays. This ensures a very small error for short periods of time, which increases as they change. To reduce the effect of those changes, time correlation of major error sources is increased through the use of external correction data, such as precise ephemeris, high-rate clock corrections and ionospheric TEC maps [4]. This reduces the error growth, increasing the time of sub-decimeter accuracy. One advantage of this method is that it does not require a reference station. However, since most of the external corrections are not available in real time, the positioning is only obtained with post-processing.

The method proposed here combines the ambiguity cancellation concept of the time-differences approach with mitigation of error sources by means of a near reference station such as in RTK. This way, the final accuracy is expected to remain higher than float RTK, but worse than the fixed RTK single-frequency solutions and will depend most on satellite geometry, decreasing in time much slower than as in [2]-[3].

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II PROPOSED METHOD

Assume that the reference receiver is located at a known fixed surveyed location near the rover receiver (also called user receiver). User initial and current positions occur at times t_i and t_j, respectively. The main goal is to estimate the user displacement vector from t_i to the current time t_j. Suppose that during the interval $[t_i, t_j]$ no cycle slip occurs for a maximum of n common satellites tracked by both receivers. Measurements of code (pseudoranges) and carrier phase, as well as satellite ephemeris and receiver related parameters (time tags, cycle slip flags, signal strength, etc.) from both receivers at both epochs t_i and t_j are sent to a central processing function, which computes the desired user displacement vector as well as error estimates.

Observation Model

Based on usual pseudorange and carrier phase measurement models [1], the original undifferenced observations vector obtained from the two receivers can be written for satellite *Si* and epoch *t*, in this case, as

$$O^{Si}(t) = \begin{pmatrix} PR_u^{Si}(t) \\ \phi_u^{Si}(t) \\ PR_{ref}^{Si}(t) \\ \phi_{ref}^{Si}(t) \end{pmatrix} = \begin{pmatrix} \rho_u^{Si}(t)+I^{Si}(t)+T_u^{Si}(t)+dt^{Si}(t)+dt_u(t)+\varepsilon_c^{u,Si} \\ \rho_u^{Si}(t)+I^{Si}(t)+T_u^{Si}(t)+dt^{Si}(t)+dt_u(t)+\tilde{N}_u^{Si}+\varepsilon_p^{u,Si} \\ \rho_{ref}^{Si}(t)+I^{Si}(t)+T_{ref}^{Si}(t)+dt^{Si}(t)+dt_{ref}(t)+\varepsilon_c^{ref,Si} \\ \rho_{ref}^{Si}(t)+I^{Si}(t)+T_{ref}^{Si}(t)+dt^{Si}(t)+dt_{ref}(t)+\tilde{N}_{ref}^{Si}+\varepsilon_p^{ref,Si} \end{pmatrix}, \quad (1)$$

where superscripts and subscripts denote satellites and receivers, respectively. Also, *u, ref, PR, φ, I, T, dt* and $\tilde{N}$ denote user receiver, reference receiver, pseudorange, carrier-phase, ionospheric delay, tropospheric delay, clock error and non-integer ambiguity, respectively. All ρ terms mean distance from satellite at time of transmission to receiver at time of signal arrival. Note that delay terms are already in units of distance instead of time by simply multiplying them by the speed of light *c*. The non-integer ambiguity terms $\tilde{N}$ are formed by the integer ambiguity, $N.\lambda$, and initial phases of receivers and satellites, remaining constant as long as no cycle slips occur. The remaining ε error terms are formed by measurement errors and non-modeled error sources, such as multipath, ionosphere gradient and ephemeris errors, and are supposed to have zero average, variance σ_{PR}^2 and σ_ϕ^2 for code and phase measurements, respectively, and be uncorrelated to each other. Since in (1) there are 4 scalar observations per satellite and epoch, there are overall $8.n$ observation scalars for the n common satellites tracked by both receivers at both epochs.

Since most of the unknowns are of no interest, single differencing the observations with respect to the receivers, shown in (2), has the advantage of eliminating satellite clock errors. The equivalence property for differenced and undifferenced observations [1] guarantees that this procedure will lead to the same results as the undifferenced approach, even though the variance of the error terms ε_{SD} doubles as a result of the subtraction of the random variables in the original observations. By using the original observations, some of the unknowns would exhibit degrees of freedom, characterized by a rank-deficient design matrix, turning its treatment more difficult. Single differencing also eliminates that need by reducing the number of unknowns as some of them can be grouped as shown in (2).

$$SD^{Si}(t) = \begin{pmatrix} \Delta PR_{u,ref}^{Si}(t) \\ \Delta\varnothing_{u,ref}^{Si}(t) \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} . O^{Si}(t)$$

$$= \begin{pmatrix} [\rho_u^{Si}(t)-\rho_{ref}^{Si}(t)]+[T_u^{Si}(t)-T_{ref}^{Si}(t)]+ +[dt_u(t)-dt_{ref}(t)]+\varepsilon_{c,SD}^{u,Si} \\ [\rho_u^{Si}(t)-\rho_{ref}^{Si}(t)]+[T_u^{Si}(t)-T_{ref}^{Si}(t)]+ +[dt_u(t)-dt_{ref}(t)]+[\tilde{N}_u^{Si}-\tilde{N}_{ref}^{Si}]+\varepsilon_{c,SD}^{u,Si} \end{pmatrix} \quad (2)$$

$$= \begin{pmatrix} \Delta\rho_{u,ref}^{Si}(t)+\Delta T_{u,ref}^{Si}+\Delta dt_{u,ref}(t)+\varepsilon_{c,SD}^{u,Si} \\ \Delta\rho_{u,ref}^{Si}(t)+\Delta T_{u,ref}^{Si}+\Delta dt_{u,ref}(t)+\Delta\tilde{N}_{u,ref}^{Si}+\varepsilon_{c,SD}^{u,Si} \end{pmatrix}$$

Here, $\Delta\rho_{u,ref}^{Si}(t)$ is a function of user, reference and satellite positions, $X_u(t)$, X_{ref} and $X_{Si}(t)$, respectively. $\Delta T_{u,ref}^{Si}$ is a function of $X_u(t), X_{ref}, X_{Si}(t)$ and tropospheric parameters. $\Delta dt_{u,ref}(t)$ and $\Delta\tilde{N}_{u,ref}^{Si}$ are scalar unknowns.

Note that the $I^{Si}(t)$ terms in (1) do not depend on the receiver and were eliminated in (2) by the single differencing as well. This is based on the common assumption that user and reference station are close to each other. As they separate, the residual error terms will increase reducing the accuracy. This reflects the RTK-like component of the proposed method in contrast to the time differences approach, where atmospheric errors tend to grow with increasing time.

To achieve the proposed accuracy, unlike ionospheric delays that are difficult to model, tropospheric delays must be consistently modeled. For that reason the T terms depend both on satellite and receiver. Since our goal is decimeter-level accuracy, a simple model should work fine, such as Hopfield with most parameters fixed at site-average values and a few more significant ones being included as unknowns, for example.

For direct extraction of the user displacement vector, $X_u(t_i,t_j)=X_u(t_j)-X_u(t_i)$, it is convenient to define $X_u(t_i,t_j)$ as one of the unknowns and use $X_u(t_i)$ as another, substituting $X_u(t_j)$ by $X_u(t_i) - X_u(t_i, t_j)$. This way, there are two three-dimensional vector unknowns to solve for. The other unknowns are the single-differenced clock errors at both epochs, $\Delta dt_{u,ref}(t_i)$ and $\Delta dt_{u,ref}(t_j)$, float ambiguities $\Delta \tilde{N}^{Si}_{u,ref}$ for all n satellites in view, and m tropospheric parameters for each epoch (t_i and t_j). Therefore, there are a total of $2.3+2+2m+n = 8+2m+n$ unknowns to be solved for, but only $X_u(t_i, t_j)$ is of interest. Since there are $4n$ SD observations (half of the 8n undifferenced observations), if one has $m=1$, then there must be at least 4 satellites in view in this case.

Linearization of the Observation Model

For application of the least squares method it is first necessary to linearize the non-linear observation model with respect to the unknowns. This process is largely employed in literature (see, e.g., [1]) and leads to the linearized equation

$$L = AX + V, \tag{3}$$

where L is known as the (linearized) observable vector, A is the Jacobian or design matrix, X is the unknown vector and V is the residual vector. Generally, this equation must be solved a number of times through least squares method until the process converges.

Weighted Least squares adjustment

To apply the well-known weighted least squares method [1] to the linearized observation model, first we form the normal equation

$$(A^T PA)X = A^T PL\ , \tag{4}$$

where P is a symmetric and definite weight matrix, typically chosen as the inverse of the covariance matrix of L for a BLUE (Best Linear Unbiased Estimator). Solving (4) we obtain X for the iteration loop. After convergence, the unknowns can finally be computed, and hence the desired displacement vector.

The quality of the resulting adjusted unknown parameters in terms of their covariance matrix is obtained from the expression

$$\Sigma_{\hat{X}} = \sigma_0^2 (A^T PA)^{-1}. \tag{5}$$

It is important to notice that the complete procedure described here is intended to be a first approach to solve the proposed problem for only one point besides the starting position. Although not the optimal way in terms of computational effort, complete trajectory determination can be done through consecutive such procedures. There are obviously more efficient ways of computing trajectories, and that may be addressed in future works.

III. SIMULATION DESCRIPTION

In order to evaluate the proposed method for TRDP, a simplified two-dimensional simulation of an environment containing the main aspects of a satellite navigation system was performed. The simplified geometry adopted is schematized in Fig.1. All satellites are uniformly spaced at radius 26600km and rotate angularly at two revolutions per day, which illustrates some changing geometry, but satellite movement during signal travel time is not considered for simplicity. Other parameters are chosen consistently with realistic approximate values, such as earth radius of 6400km, ionospheric delays ranging from 1 to 20 meters, tropospheric refractivity of around 5.10^{-5}, reference station to user distance not greater than 100km, σ_{PR}'s of one meter and σ_{Φ}'s of one centimeter.

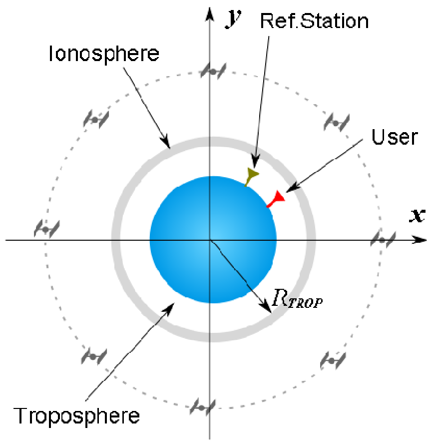

Fig.1. Simulation assumed geometry.

Common satellites tracked by both receivers during all epochs are chosen by an elevation masking angle of 15º. The tropospheric delay model is as simple as

$$T^{Si}_{rec}(t) = N_T(t).\left(R_{TROP} - \sqrt{x^2_{rec} + y^2_{rec}} \right).\frac{1}{\sin(Elev(rec, Si))}, \tag{6}$$

where *rec* denotes the receiver, *u* or *ref*, N_T (t) is the local refractivity index at time t, R_{TROP} is the troposphere outer radius considered at 50km above earth's surface, and *Elev(rec,Si)* is the elevation angle of satellite *Si* as seen by receiver *rec* with respect to the local earth's horizontal plane. Note that the only unknown parameter is $N_T(t)$, since R_{TROP} is supposed given, therefore $m = 1$. This simple model tends not to be very realistic for low elevation angles, but since an elevation mask of 15 degrees is being used, this is reasonable enough for illustrating tropospheric effects.

After single-difference observations are formed, the TRDP problem is resolved by the procedure described in the previous item. This simulation was implemented in Matlab and the results are shown in the following section.

IV. RESULTS

To illustrate the usage of the proposed method as a trajectory determination technique, the simulation described on the previous item was repeated in a nested loop with varying satellite and user parameters from epoch to epoch, as shown in Fig.2. The user describes a counterclockwise circle in a period of 30 minutes. To include tropospheric refractivity variations from initial time t_i to current time t_j, these were generated randomly as uniform distributed in the range $[45, 55].10^{-6}$ and uncorrelated to each other. Clock error was generated in a similar way. The number of satellites in view was always n = 6.

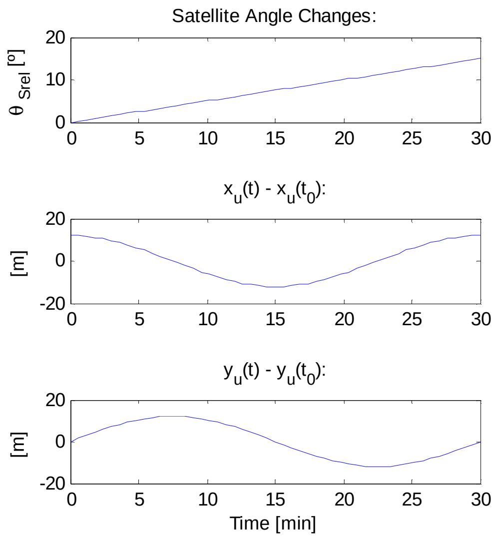

Fig.2. Satellite belt angle and user trajectories used for simulation.

The results from a single well representative outcome of the simulation program are shown in Figs. 3-5. Comparing Fig. 3 and Fig. 4, the effectiveness of the method as a relative trajectory determination tool is reinforced by the better preserved circle shape. Finally, Fig. 5 shows both the estimated and actual 2D errors of the relative trajectory in time. Sub metric accuracy was achieved for at least 10 minutes, with increasing errors being caused mostly by the satellite geometry change. Fig. 5 indicates the consistency of the estimated and actual errors as well.

V. CONCLUSIONS AND FURTHER WORK

This work was intended to be a first approach to solve the proposed TRDP problem in a simple simulation environment. Although many simplifying suppositions were made, the obtained results were promising. The next step in evaluating its effectiveness is to perform further tests using more realistic three-dimensional models and real GNSS data.

Limitations of the present work include not considering the signal propagation time, ephemerides and error models, which should be carefully addressed in further developments. The main limitations of the method are the necessity of reference station and cycle slip vulnerability. Advantages include the possibility of real-time implementation, slower error growing than the time-differences approach proposed by [2]-[3] and not having the need to solve for ambiguities.

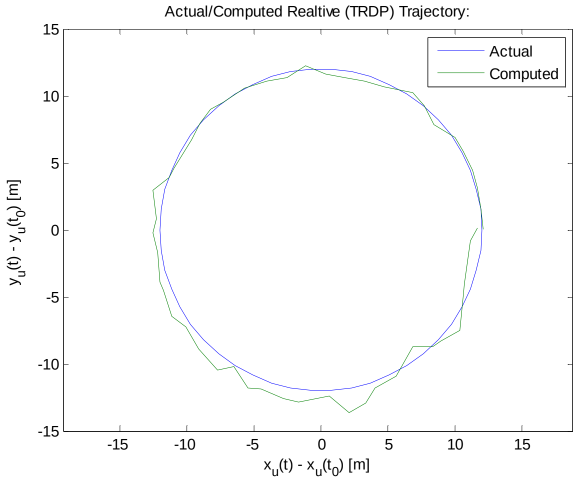

Fig.3. Comparison of actual and computed trajectory in TRDP sense.

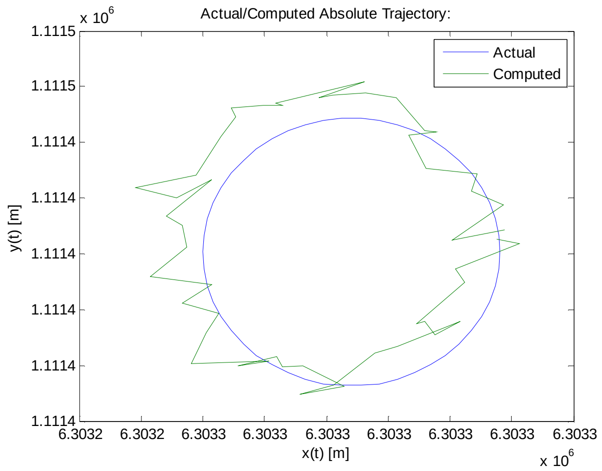

Fig.4. Comparison of actual and estimated trajectory of absolute user position.

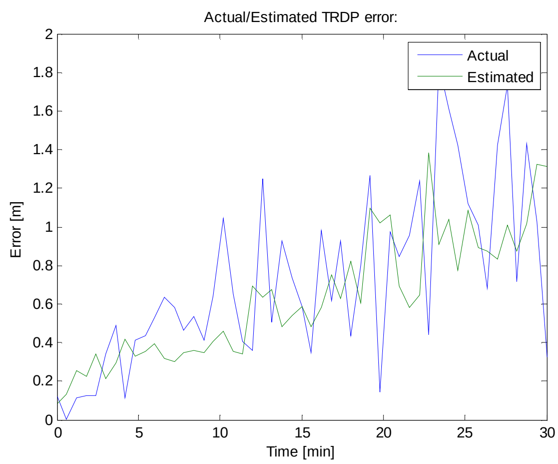

Fig.5. Actual and estimated TRDP error versus time.

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